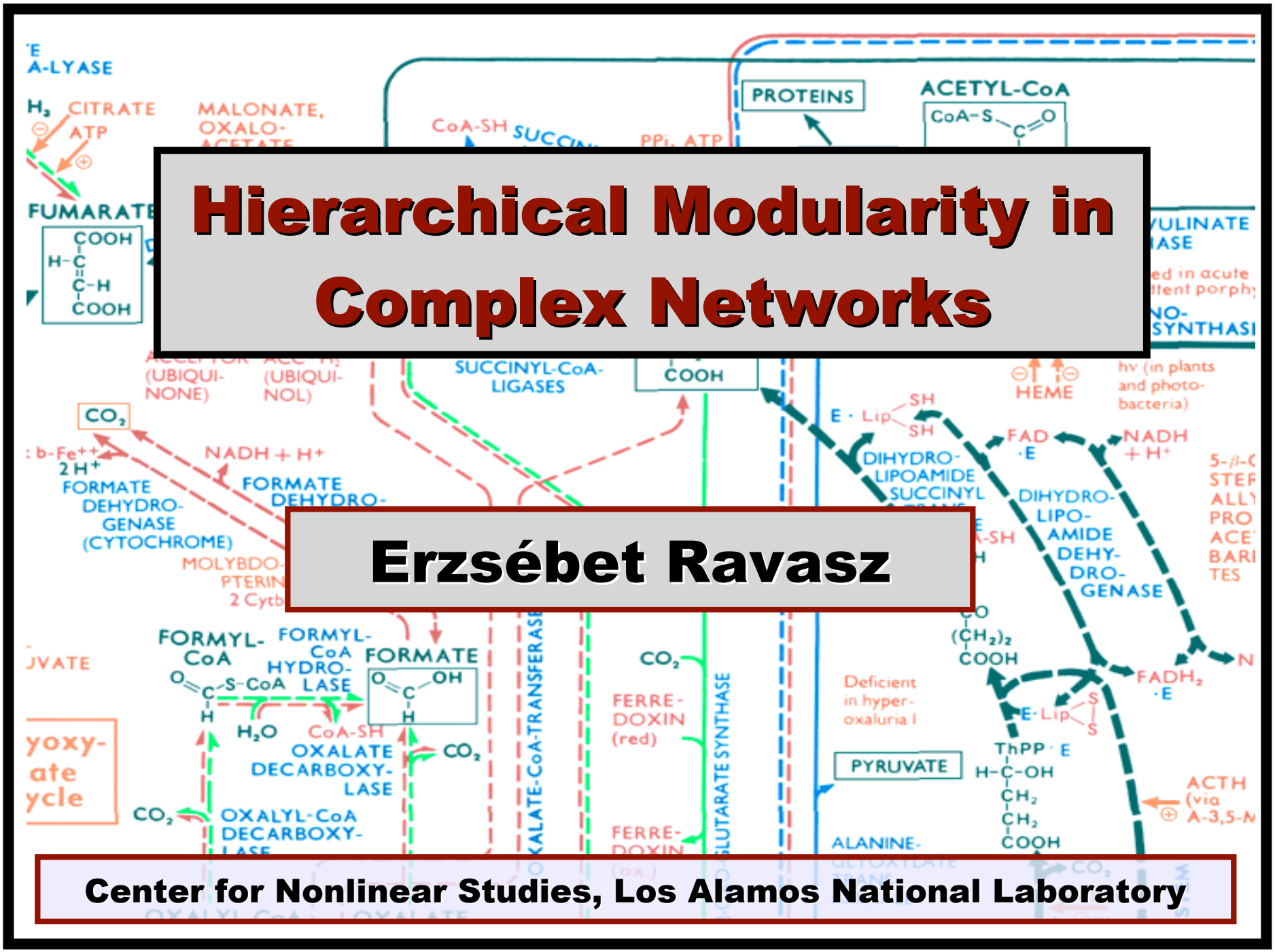
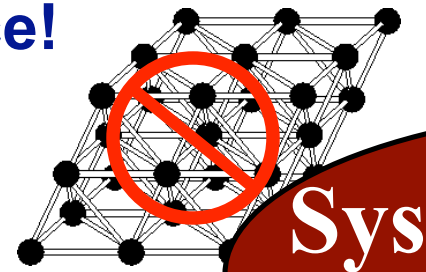
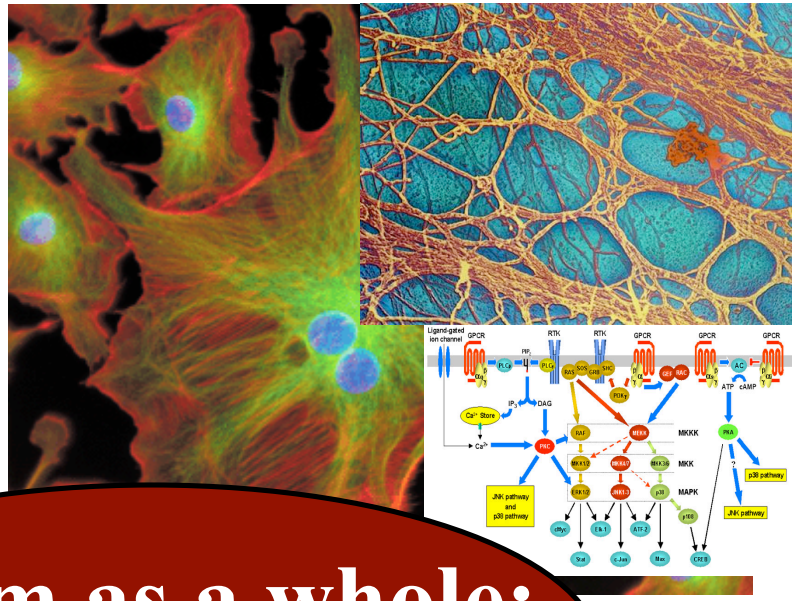

Hierarchical Modularity in Complex Networks
Erzsébet Ravasz
Center for Nonlinear Studies, Los Alamos National Laboratory

Networks?

The cell is not regular

→ Many different components
→ A variety of interactions
→ Not a lattice!

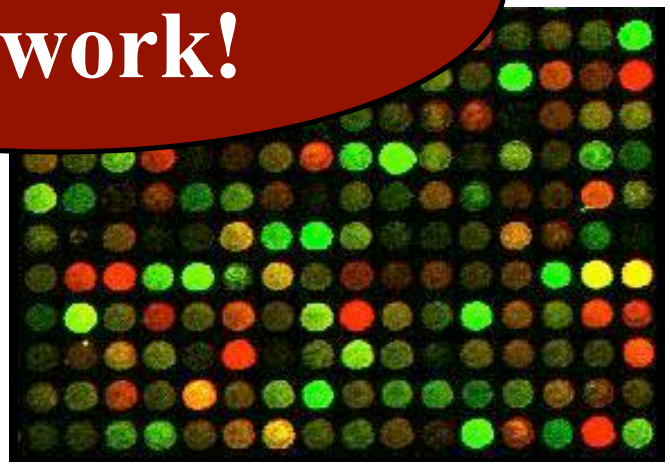




System as a whole: network!

Systems Biology

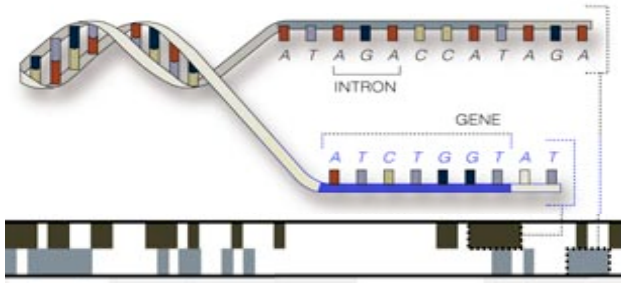
→ Data on system level
→ Methods and focus shift
→ Asking for statistical physics

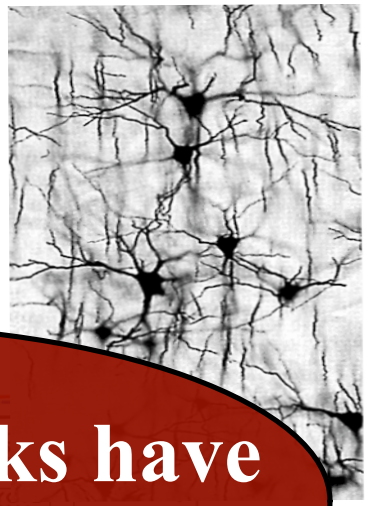


Networks in life

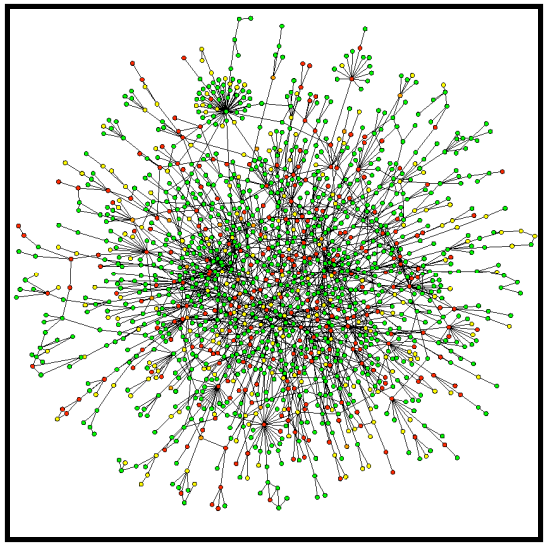
Biology

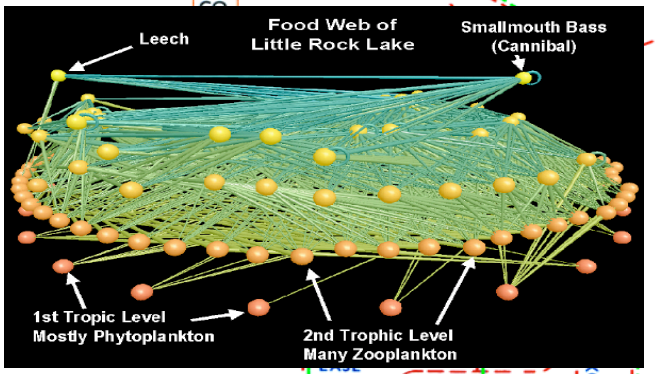
- → Genetic regulation
- → Protein-protein interactions
- → Metabolic networks
- → Neuron networks
- → Food webs

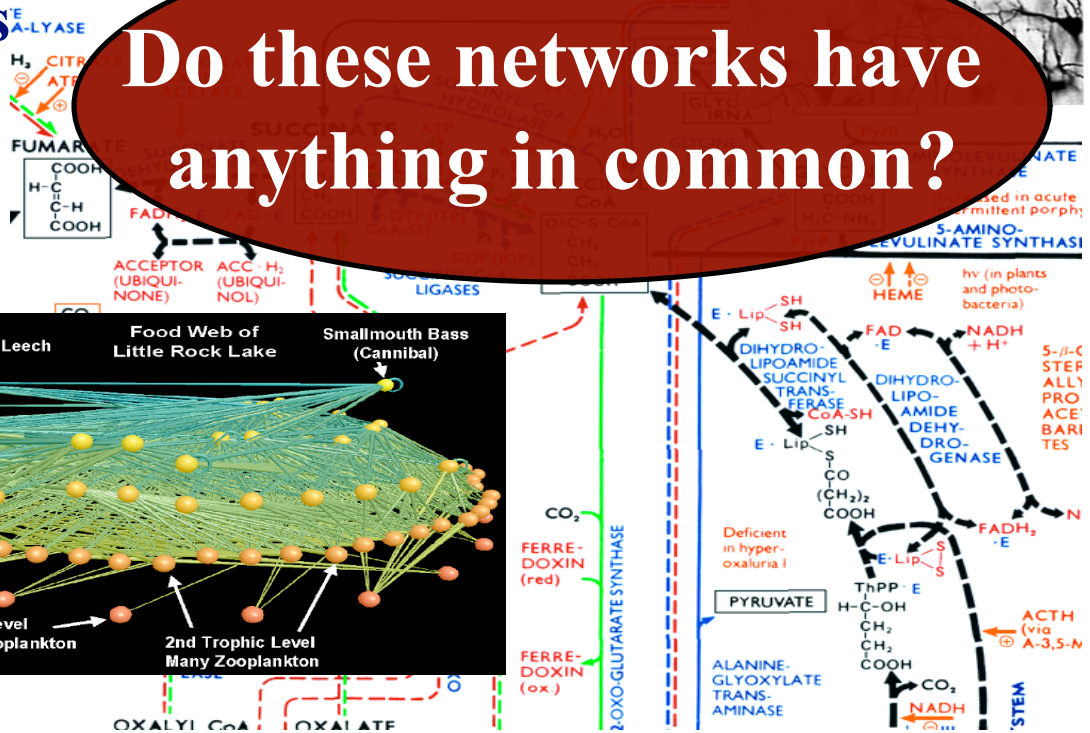




Do these networks have anything in common?



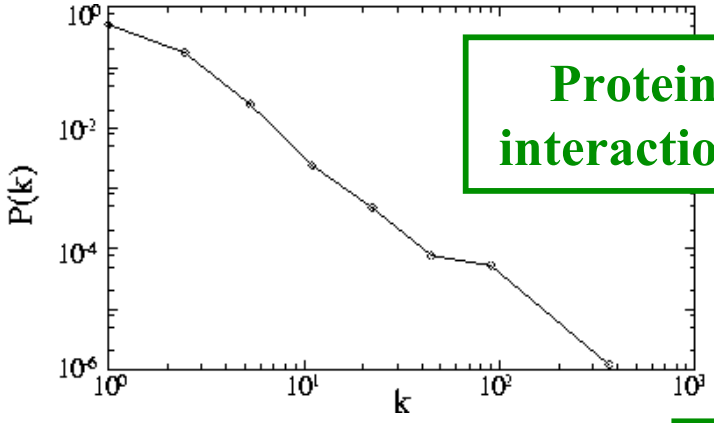




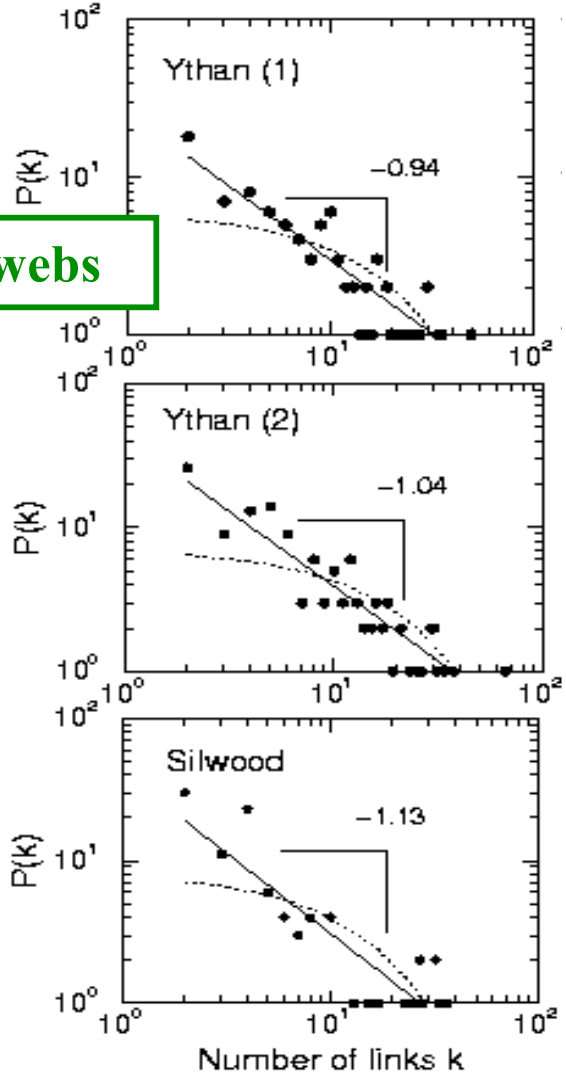
Degree distribution

Not random networks!

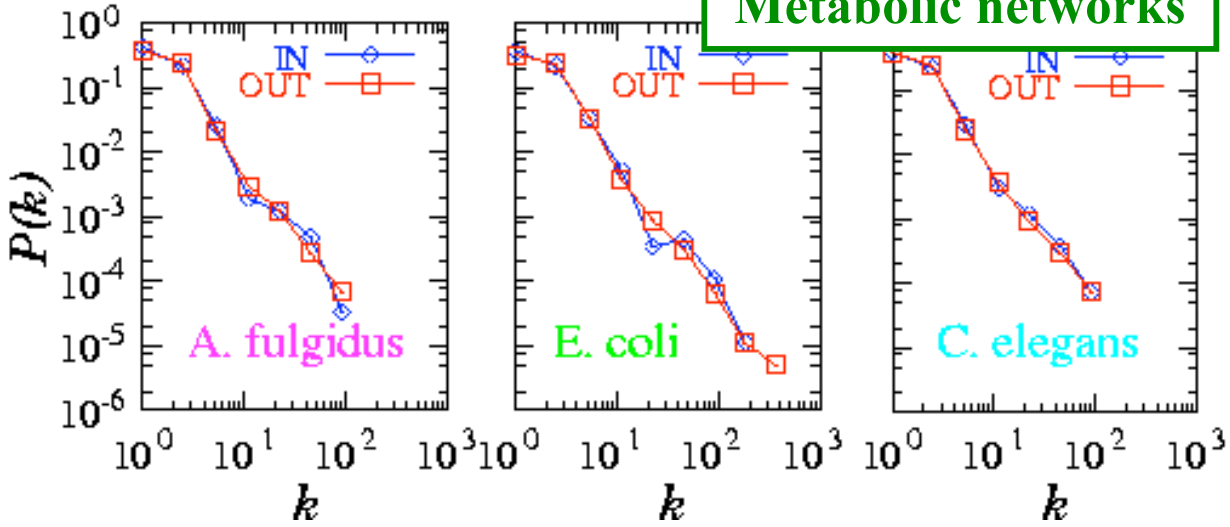
Protein interactions



Food webs



Metabolic networks

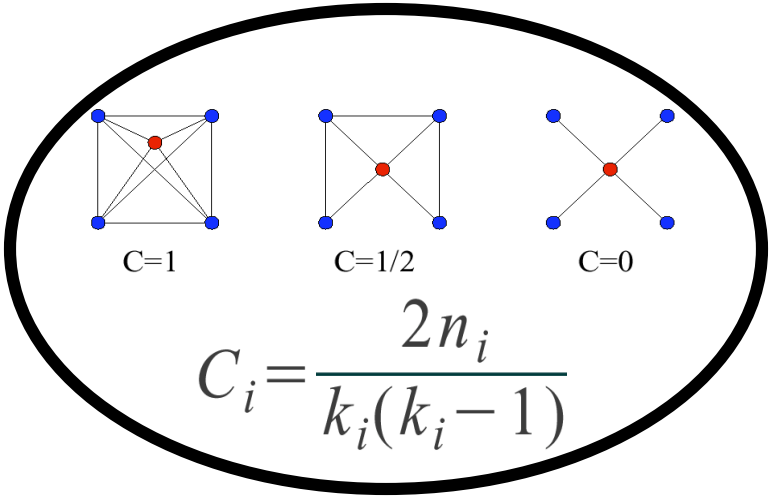


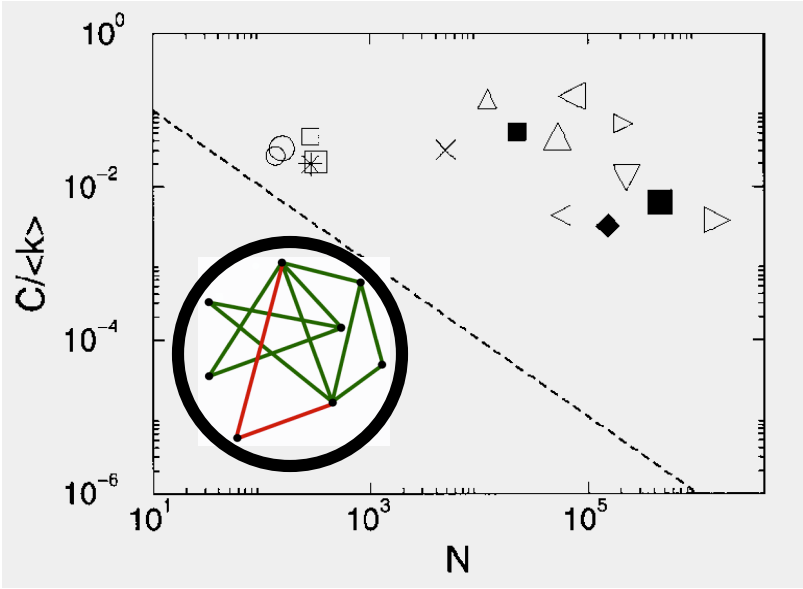
Clustering

Clustering coefficient

→ High average in real networks

→ Scale-free model: $C \sim (\ln N)^2/N$





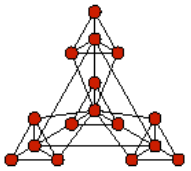
Modular organization

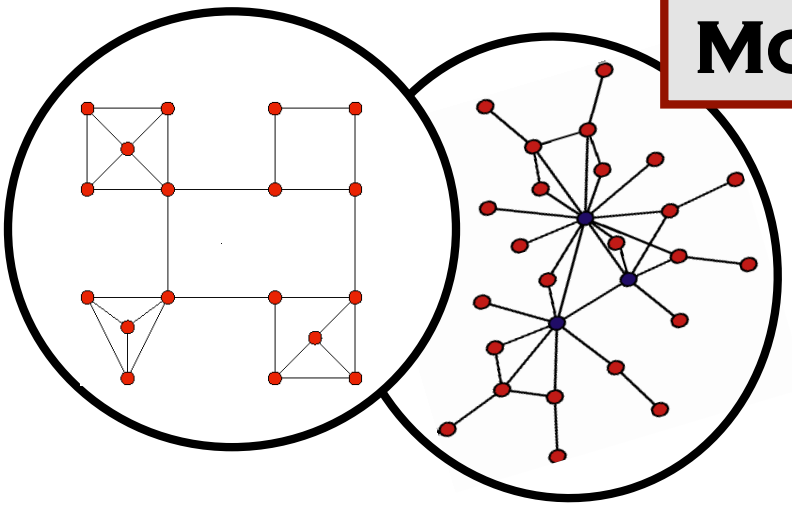
→ protein complexes

→ regulatory modules

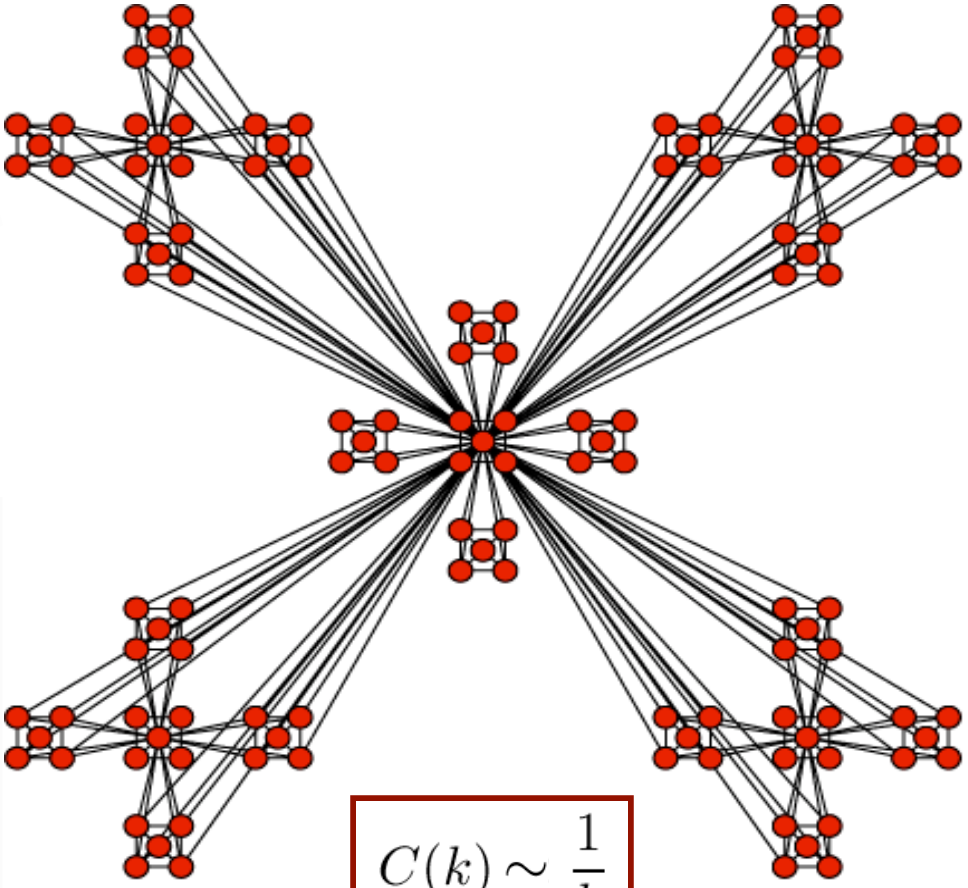
→ pathways

Module hierarchy

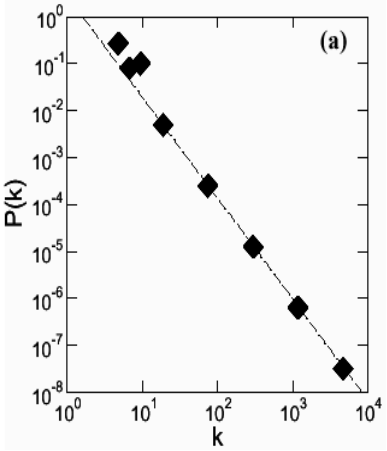


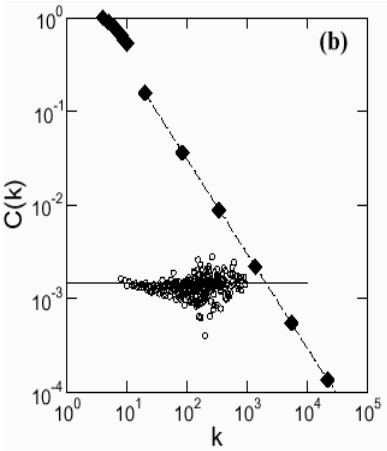


Modular And Scale-free?



Hierarchical model

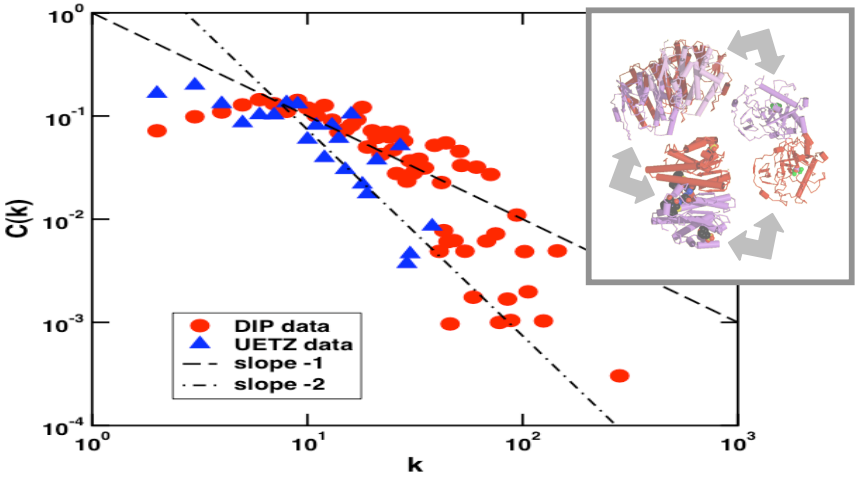




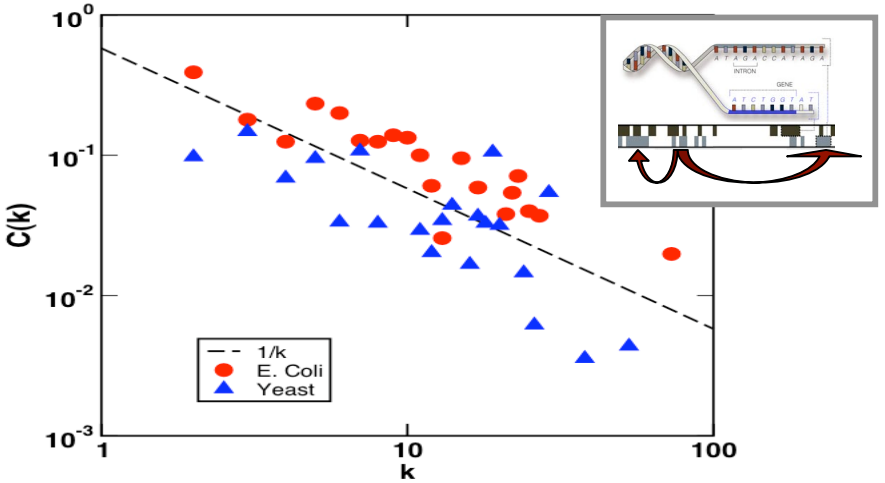
$$C(k) \sim \frac{1}{k}$$

Hierarchy in Biology

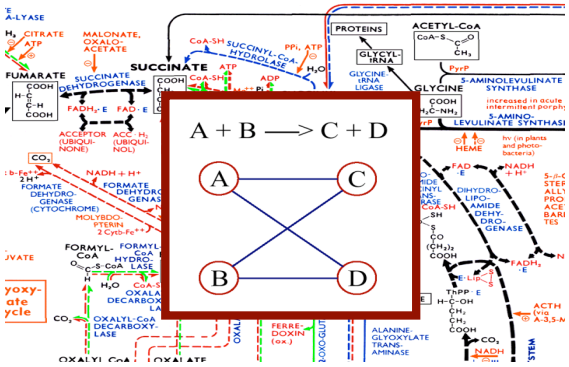
Protein-protein interaction

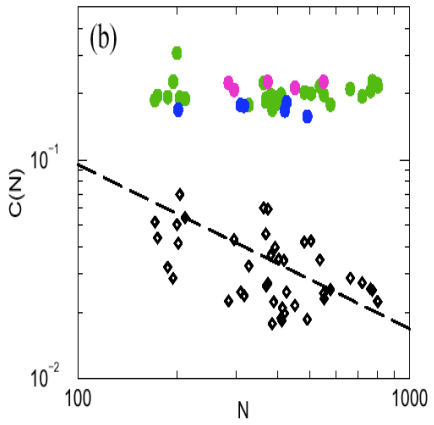


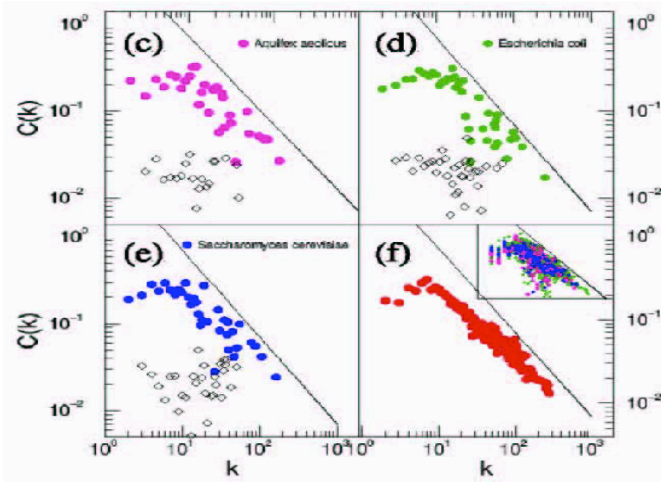
Regulatory networks



Metabolic networks

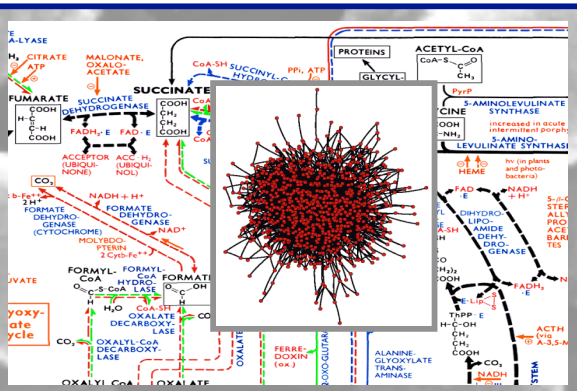


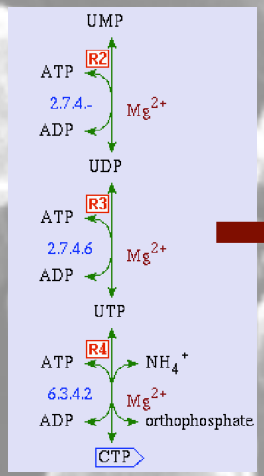


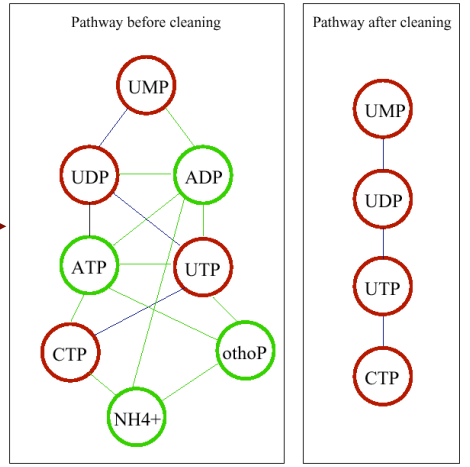


The metabolism of *E. Coli*

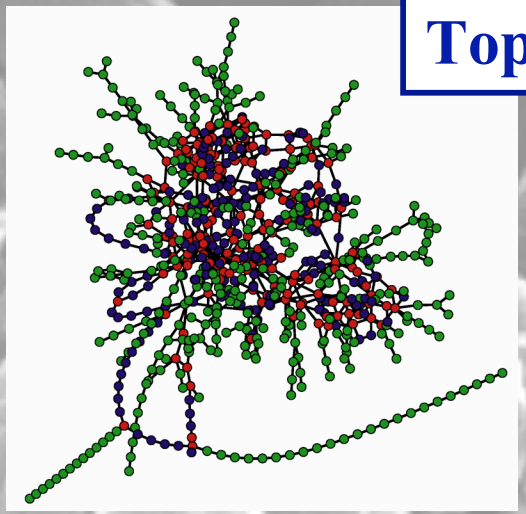
Biochemical reduction

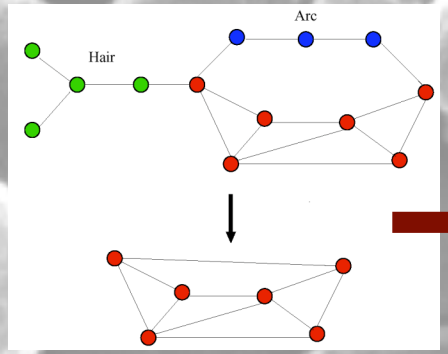


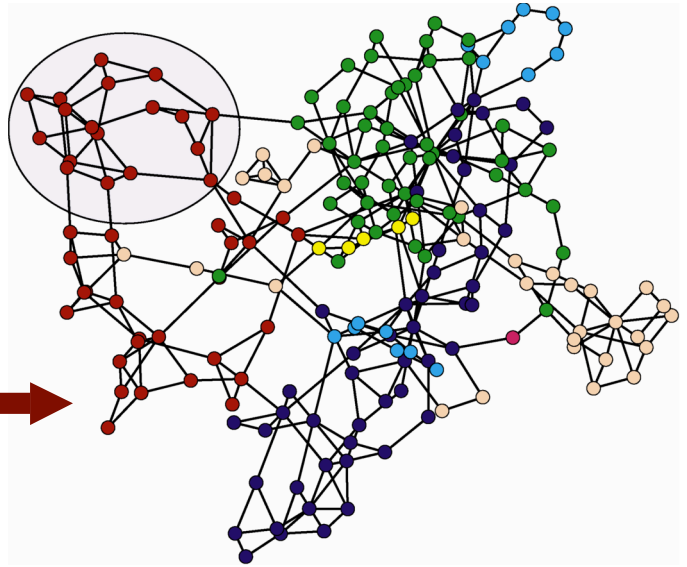




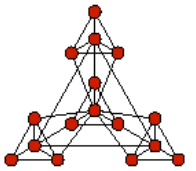
Topological reduction





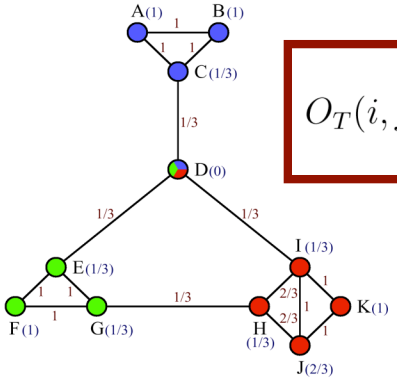


Finding the modules



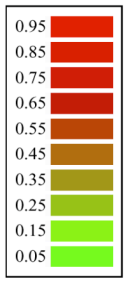
Hierarchical clustering

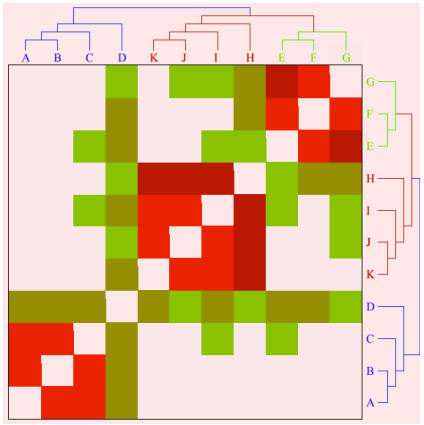
→ Similarity matrix

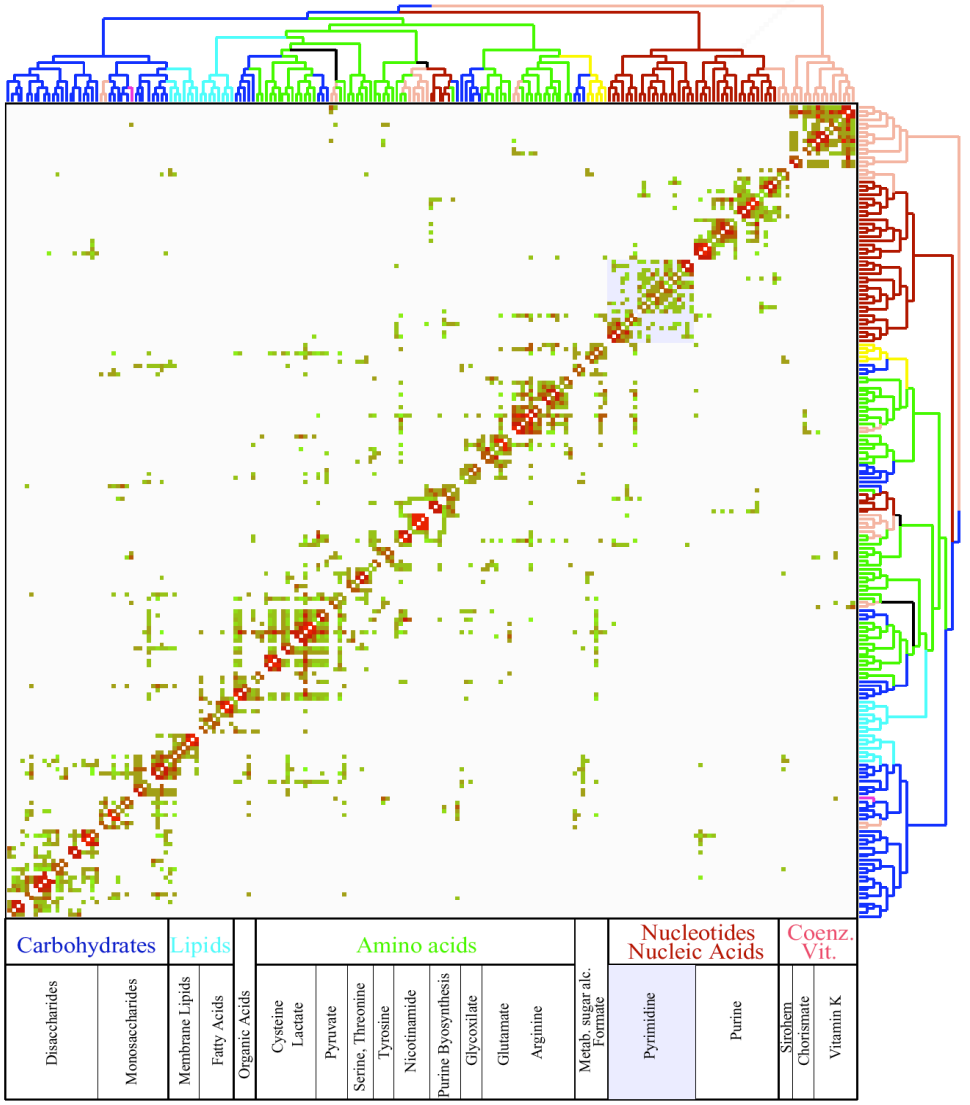


$$O_T(i,j) = \frac{\sum_{l=1}^{N}{}' l_{i,l} \cdot l_{j,l} + l_{i,j}}{\min(k_i, k_j) + 1 - l_{i,j}}$$

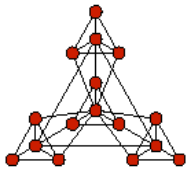
→ Average linkage clustering (UPGMA)



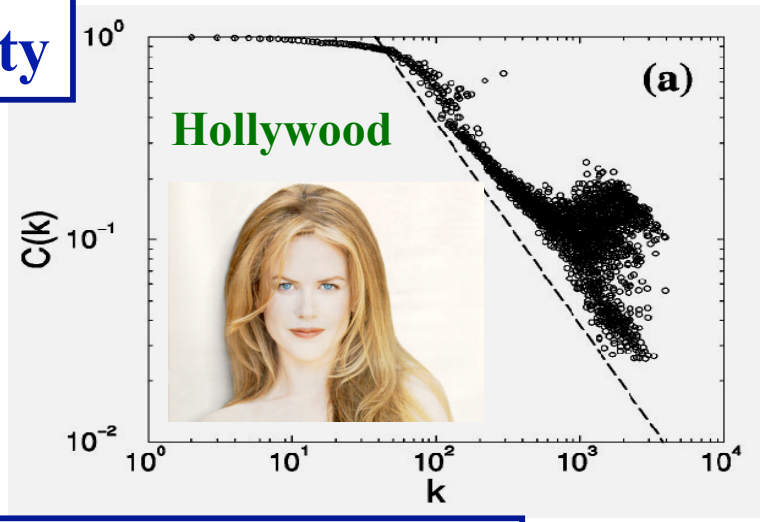




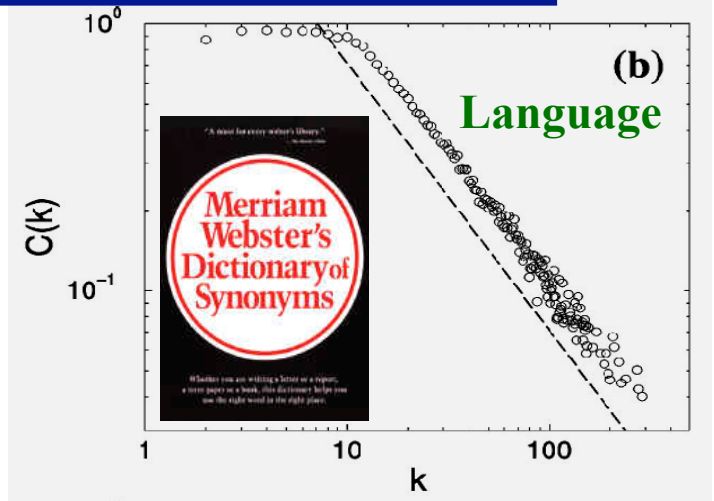
Hierarchical networks



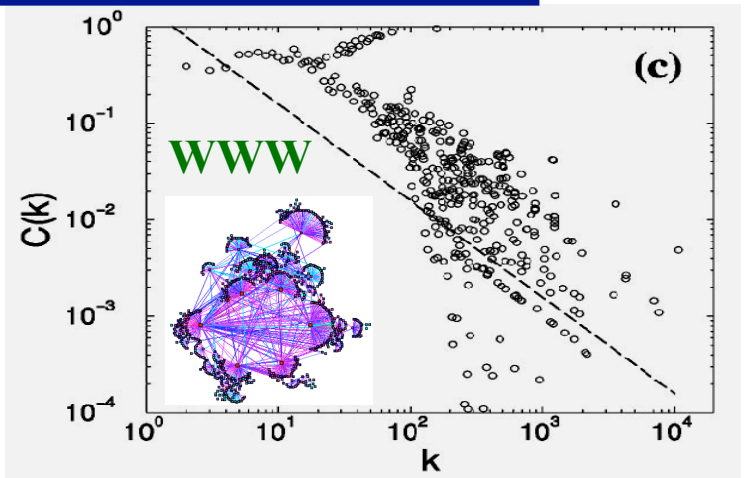
Society

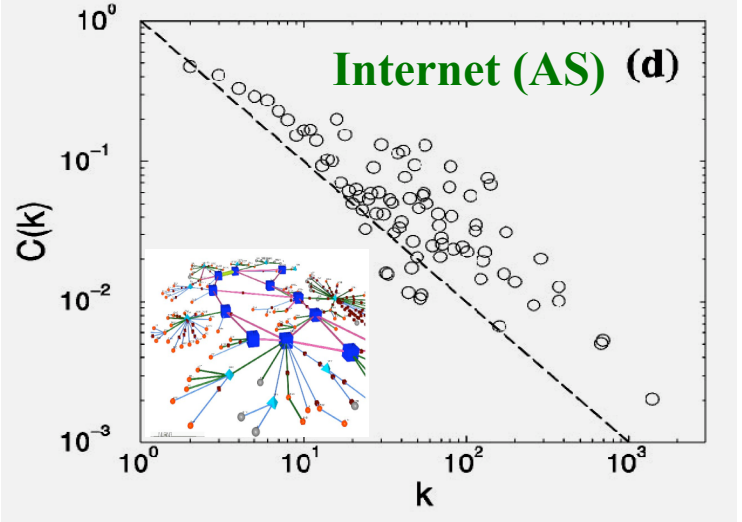


Human communication



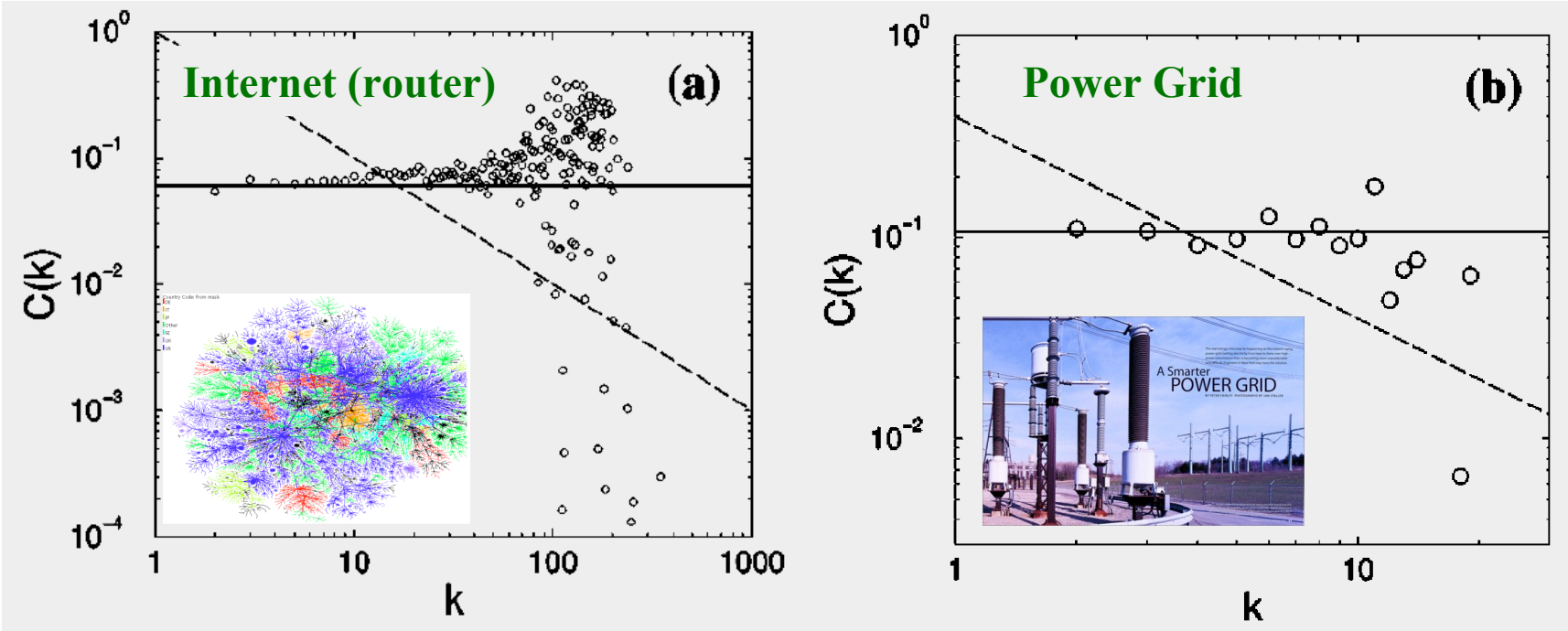
The electronic skin





Absence of hierarchy

Geographically localized networks



Economic pressure to minimize link lengths

Los Alamos NATIONAL LABORATORY EST.1943

Thank you!

http://www.nd.edu/~networks

Thanks to:

- Albert László Barabási
- Zoltán Oltvai